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Information

Professional Services Committee

Discussion of the Foundational Mathematics Authorization and Associated Subject Matter Requirements for the Authorization

Executive Summary: This agenda item presents information about the authorization scope of the current Foundational-level Mathematics credential and the associated subject matter requirements for candidates to meet the subject matter requirement for this credential. The discussion includes issues and concerns regarding the authorization scope raised by mathematics faculty members and the California Association of Mathematics Teacher Educators (CAMTE).

Policy Question: Does the Single Subject Foundational-Level Mathematics credential appropriately authorize an individual to teach the specified public school mathematics classes, and is the current authorization appropriately supported by the associated Subject Matter Requirements?

Recommended Action: For information only

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Strategic Plan Goal

I. Educator Quality

- Maintain expectations for educator preparedness and performance that are responsive to the needs of California's diverse student population and promote 21st century teaching and learning.

Discussion of the Foundational-Level Mathematics Authorization and Associated Subject Matter Requirements for the Authorization

Introduction
This agenda item presents information about the authorization scope of the current Single Subject Foundational-Level Mathematics credential, and examines whether this authorization appropriately supports the assignment of holders of this credential to public school mathematics courses. In addition, the item presents concerns about the authorization scope raised by mathematics faculty members and the California Association of Mathematics Teacher Educators (CAMTE).

Background
Over the past four years, staff has presented agenda items to the Commission addressing issues in teacher preparation related to teaching mathematics, including subject matter preparation, professional preparation, the credentials, authorizations, and the array of courses authorized to be taught by the different authorizations. The recent adoption of the Common Core State Standards for California has increased attention on the content of all of the common core subject areas, including mathematics.

Following a December 2008 Commission discussion regarding the Mathematics Specialist credential (http://www.ctc.ca.gov/commission/agendas/2008-12/2008-12-3G.pdf), an advisory panel was convened by the Executive Director to update and revise the standards for Mathematics Specialist programs. As part of the work of the panel, there was some discussion of panel member concerns regarding the alignment between the subject matter requirements (SMRs) for the Foundational-Level Mathematics credential and the content of the K-12 coursework that a holder of that credential was authorized to teach, but the panel did not address this issue further. It is appropriate now to take a closer look at the Foundational-Level mathematics preparation and authorization scope to address issues and concerns that have arisen over time.

Overview of Mathematics Authorizations for the Secondary Level
The (full) **single subject Mathematics credential** authorizes an individual to teach every level of mathematics from grades K-12 in departmentalized settings. More specifically, the single subject mathematics credential authorizes holders to teach mathematics courses including Algebra I, Geometry, Algebra II, Trigonometry, Probability and Statistics, Introductory Analysis, Calculus.

The authorization for the single subject **Foundational-Level Mathematics (FLM) credential** states:

> *This document authorizes the holder to teach the content areas in general mathematics, algebra, geometry, probability and statistics, and consumer*

mathematics in grades twelve and below, including preschool, and in classes organized primarily for adults.

Individuals who hold a FLM credential are clearly not authorized to teach Trigonometry, Introductory Analysis, or Calculus classes. The Commission's authorizations do not address the distinctions between specific levels of K-12 courses such as, for example, teaching remedial, advanced, or honors classes, since the definitions of such classes and the content of these classes vary widely from district to district. However, there is a distinction in terms of courses identified as Advanced Placement, since these courses follow a distinctive, commonly-accepted range and sequence of content.

The Subject Matter Requirements for Mathematics

The mathematics content that forms the basis for determining subject matter competence is defined by a set of subject matter requirements (SMRs) with respect to mathematics that beginning teachers are expected to know. Appendix A provides the list of the Mathematics SMRs for single subject teacher candidates. These SMRs form the basis for both the candidate competencies contained within the mathematics subject matter preparation program standards and the mathematics content from which CSET examination items are developed. The SMRs are aligned with the K-12 student academic content standards in Mathematics and with the applicable state framework for Mathematics. Single subject candidates who successfully complete a Commission-approved subject matter preparation program in mathematics or who pass the CSET: Mathematics examination, are deemed to be subject matter competent in mathematics for the grade levels authorized by their teaching credential.

The mathematics subject matter requirements for single subject candidates are organized into six major domains. The domains are similar to the domains indicated in the grades 8-12 student academic content standards for individual mathematics-related subjects. These major content domains are delineated further by "subdomains," or content that is more specific. For example, in the Geometry domain, the subdomains include Parallelism; Plane Euclidean Geometry, Three-Dimensional Geometry, and Transformational Geometry. According to the Handbook for subject matter preparation programs in Mathematics (http://www.ctc.ca.gov/educator-prep/STDS-subject-matter.html) candidates are required to know the following topics in depth:

Domain 1: Algebra
- Algebraic Structures
- Polynomial Equations and Inequalities
- Functions
- Linear Algebra

Domain 2: Geometry
- Parallelism
- Plane Euclidean Geometry
- Three-Dimensional Geometry
- Transformational Geometry

Domain 3: Number Theory
- Natural Numbers

Domain 4: Probability and Statistics
- Probability
- Statistics

Domain 5: Calculus
- Trigonometry
- Limits and Continuity
- Derivatives and Applications
- Integrals and Applications
- Sequences and Series

Domain 6: History of Mathematics
- Chronological and Topical Development of Mathematics

Candidates demonstrate their subject matter competence for either the full mathematics credential or the FLM credential in one of two ways: either passing the CSET: Mathematics Examination (Foundational-Level or full Mathematics Examination), or completing a Commission-approved subject matter preparation program in Mathematics. The full Mathematics credential requires a candidate, using the examination route, to pass all three CSET: Mathematics Examination subtests, whereas FLM candidates are only required to pass Subtests 1 and 2 of the CSET: Mathematics Examination, which address Domains 1-4. The complete content specifications for the CSET: Mathematics examination can be found on the CSET web page: http://www.cset. nesinc.com/PDFs/CS_mathematics_SMR.pdf.

Key Issues for Commission Consideration

A complex and interrelated series of issues has arisen over time regarding the current scope of the FLM credential authorization, the scope of the subject matter currently required for the FLM authorization, the scope of the subject matter needed to successfully instruct K-12 students in mathematics courses as they are organized in public schools today, and the content covered within subject matter preparation programs and the corresponding examination structure used by candidates who choose the examination route to demonstrate their subject matter competency for the FLM credential. These issues are discussed below.

The Current Scope of the FLM Authorization: The FLM credential was initially proposed in 2002 by a Subject Matter Advisory Panel (SMAP) in Mathematics that had been convened to review and revise the Commission's subject matter requirements and ensure alignment to the K-12 academic content standards. The FLM was developed as a way to increase the number of teachers prepared and authorized to teach introductory mathematics courses (specifically, Algebra I and Geometry) that were required for graduation from high school. The SMAP members believed that the authorization for the FLM credential would be limited to Algebra I and Geometry, and that the Probability and Statistics classes authorized by statute were to be taught only at the introductory level.

The intent of the SMAP in proposing the FLM credential was not enacted, however, when the regulations for the FLM were promulgated. Within the documentation provided as part of the regulatory development process, the reasons stated to justify the FLM credential incorporating the Foundational-Level Mathematics Subject Matter Requirements and Credential reads as follows:

> Unlike the current Mathematics subject matter area and as reflected in the authorization for this proposal, the individual seeking certification in Foundational-Level Mathematics will not be required to verify in-depth knowledge of advanced mathematics nor will they be authorized to teach in these fields.

Thus, the language incorporated in the authorization allows an individual to teach Algebra, Geometry, Probability and Statistics, without limitation as to the level of these courses. As a result, the authorization is interpreted to allow FLM credential holders to teach Algebra II and Advanced Placement Statistics, which was not the intent of the SMAP panel. The panel did not examine the associated SMRs to ensure that individuals with the FLM are adequately prepared to teach Algebra II or Advanced Placement Statistics. It is not clear that FLM credential holders are

fully prepared to teach these higher level mathematics courses, although they are currently allowed to be assigned to courses such as these based on the current authorization scope of the FLM credential.

The subject matter currently required for the FLM authorization: Currently, the subject matter preparation required for the FLM credential includes all of the same content within Domains 1-4 as is required for candidates for the full Mathematics credential.

CSET Mathematics Subtests

Subtest 1	**Subtest 2**	**Subtest 3**
Domain 1: Algebra	Domain 3: Number Theory	Domain 5: Calculus
Domain 2: Geometry	Domain 4: Probability and Statistics	Domain 6: History of Mathematics

FLM candidates are not required to pass Subtest 3 and are not held to the content of Domains 5 and 6. However, this arrangement of the subtests has resulted in the inclusion of content within Domains 1-4 that is not actually needed for the FLM authorization and in the exclusion of some content from Domain 5 and particularly from Domain 6 that would be appropriate for FLM credential candidates. An example of included content that goes beyond what is needed for the FLM credential would be Plane Euclidean Geometry and Transformational Geometry from Domain 2; and an example of excluded content that would be appropriate for the FLM credential is the History of Mathematics from Domain 6.

It is not clear, therefore, that the subject matter requirements for the FLM credential align with the actual mathematics content needed by a holder of this credential.

The scope of the subject matter needed to successfully instruct K-12 students in mathematics courses as they are organized in public schools today: Although the FLM credential was originally conceptualized as a means to authorize an individual to teach lower level mathematics courses, not including Algebra II or Advanced Placement Statistics, the current organization of mathematics courses and the content of these courses in the public schools has made the assignment of a holder of the FLM credential problematic. This situation occurs because in some districts the content of public school mathematics classes integrates a wider range of content beyond that which the holder of an FLM credential is fully prepared to teach, and also because there may be mathematics courses which may include both Algebra II and Trigonometry content within the same course, for one example.

In addition, as California prepares for full implementation of the Common Core State Standards some districts may modify their math courses. The Common Core State Standards emphasize the integration of mathematics content and propose a three year course sequence where the courses each address concepts from Algebra I, Geometry, and Algebra II within a spiral course sequence design.

Appendix B provides more information on the range of public school mathematics courses, the content covered within these courses as this relates to the current preparation level of FLM credential candidates, and course enrollment data from 2010-11.

It is not clear, therefore, that holders of the FLM credential have been sufficiently prepared to address the scope of the subject matter now covered within applicable secondary school mathematics classes, particularly integrated mathematics classes and classes organized to address the Common Core State Standards within a spiral course sequence design that include a wide range of mathematics content.

Concerns Raised by the Mathematics Community

The California Association of Mathematics Teacher Educators (CAMTE) has expressed concerns about the Foundational Level Mathematics credential. Two letters from CAMTE are provided in Appendix C. CAMTE has raised the issue that in its view the SMRs applicable to CSET Subtests 1 and 2 do not align appropriately with the classes being taught by FLM credential holders, in part because some of the SMRs address content beyond that which FLM credential holders should be required to know. CAMTE states that as a result, some candidates have difficulty passing the CSET containing the more advanced material which the candidates should not otherwise be required to master to qualify for the FLM credential. At the same time, however, CAMTE suggests that there are other areas of mathematics that are not but should be included in the subject matter requirements for the FLM.

Focus Areas for Further Information

Two key areas of focus from the discussion presented above illustrate where further information would be helpful for future Commission consideration of these issues. The two areas are:

- What should the Foundational Mathematics credential authorize, given the original intent for this authorization, the recent adoption of the Common Core State Standards, and the variation in the types and content of mathematics courses offered in the public schools; and
- Does the current set of SMRs used by subject matter preparation programs and within the corresponding CSET examination structure appropriately reflect the intended FLM authorization, or do these SMRs need review and reorganization?

Next Steps

To provide additional input into the discussion, it would be helpful to gather additional information about what the needs of employers are with respect to teachers with a FLM authorization. It would also be helpful to find out what other members of the educator preparation and Mathematics community believe is appropriate for the FLM authorization. To that end, staff could:

(1) Survey employers (generally Human Resources or Personnel directors in districts) about their experience with FLM credentialed teachers and what courses they believe the FLM should be authorized to teach. In addition, the survey could ask what mathematics classes the employers have difficulty staffing with appropriately credentialed teachers.

Responses could include these possible authorization options for the FLM credential:

- General Math through Algebra I (K-Algebra I)

- General Math through Introductory Algebra, Geometry, and Probability and Statistics, clarifying that for individuals earning the credential in the future the FLM does not authorize the teaching of Algebra II or Advanced Placement Statistics
- General Math through Algebra I, Geometry, and Algebra II, given that the Common Core State Standards integrate this content into a sequence of integrated mathematics classes.

(2) Work with faculty from the mathematics subject matter preparation programs, the mathematics professional preparation programs, and mathematics credential holders (FLM and full mathematics) to review the SMRs for alignment with the identified authorization.

(Note: If the FLM authorization were to be modified in response to any of the issues and factors discussed above, the content and structure of the associated subject matter requirements, as well as the content and structure of the current CSET subtests based on the SMRs would need to be reviewed and revised to support the modified FLM authorization.)

Next Steps

Based on Commission discussion and possible direction to staff, staff could work with employers and other stakeholders as described above, and additional agenda items could be prepared for future Commission consideration.

Appendix A

SUBJECT MATTER REQUIREMENTS FOR PROSPECTIVE TEACHERS OF MATHEMATICS

Part I: Content Domains for Subject Matter Understanding and Skill in Mathematics

Domain 1. Algebra

Candidates demonstrate an understanding of the foundations of the algebra contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of algebra and its underlying structures, candidates have a deep conceptual knowledge. They are skilled at symbolic reasoning and use algebraic skills and concepts to model a variety of problem-solving situations. They understand the power of mathematical abstraction and symbolism.

1.1 Algebraic Structures

a. Know why the real and complex numbers are each a field, and that particular rings are not fields (e.g., integers, polynomial rings, matrix rings)
b. Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if $a < b$ and $c < 0$, then $ac > bc$)
c. Know that the rational numbers and real numbers can be ordered and that the complex numbers cannot be ordered, but that any polynomial equation with real coefficients can be solved in the complex field

(Mathematics Content Standards for California Public Schools, Grade 6, Number Sense: 1.0, 2.0; Grade 7, Algebra and Functions: 1.0; Algebra I: 1.0, 3.0-7.0, 9.0-15.0, 24.0, 25.0; Geometry: 1.0, 17.0; Algebra II: 1.0-8.0, 11.0, 24.0, 25.0; Trigonometry: 17.0; Mathematical Analysis: 2.0; Linear Algebra: 9.0, 11.0)

1.2 Polynomial Equations and Inequalities

a. Know why graphs of linear inequalities are half planes and be able to apply this fact (e.g., linear programming)
b. Prove and use the following:
   - The Rational Root Theorem for polynomials with integer coefficients
   - The Factor Theorem
   - The Conjugate Roots Theorem for polynomial equations with real coefficients
   - The Quadratic Formula for real and complex quadratic polynomials
   - The Binomial Theorem
c. Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra

(Mathematics Content Standards for California Public Schools, Grade 7, Algebra and Functions: 2.0-4.0; Algebra I: 1.0, 2.0, 4.0-10.0, 12.0-15.0, 17.0-23.0; Algebra II: 2.0-11.0, 16.0, 17.0; Trigonometry: 17.0, 18.0; Mathematical Analysis: 4.0, 6.0)

1.3 Functions

a. Analyze and prove general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions)
b. Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems)
c. Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems)

(Mathematics Content Standards for California Public Schools, Grade 6, Algebra and Functions: 1.0; Grade 7, Number Sense: 1.0, 2.0; Algebra and Functions: 3.0; Algebra I: 3.0-6.0, 10.0, 13.0, 15.0-18.0, 21.0-23.0; Algebra II: 1.0-4.0, 6.0-17.0, 24.0, 25.0; Trigonometry: 2.0, 4.0-8.0, 19.0; Mathematical Analysis: 6.0, 7.0; Calculus: 9.0)

1.4 Linear Algebra

a. Understand and apply the geometric interpretation and basic operations of vectors in two and three dimensions, including their scalar multiples and scalar (dot) and cross products
b. Prove the basic properties of vectors (e.g., perpendicular vectors have zero dot product)
c. Understand and apply the basic properties and operations of matrices and determinants (e.g., to determine the solvability of linear systems of equations)

(Mathematics Content Standards for California Public Schools, Algebra I: 9.0; Algebra II: 2.0; Mathematical Analysis: 1.0; Linear Algebra: 1.0-12.0)

Domain 2. Geometry

Candidates demonstrate an understanding of the foundations of the geometry contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of geometry and its underlying structures, candidates have a deep conceptual knowledge. They demonstrate an understanding of axiomatic systems and different forms of logical arguments. Candidates understand, apply, and prove theorems relating to a variety of topics in two- and three-dimensional geometry, including coordinate, synthetic, non-Euclidean, and transformational geometry.

2.1 Parallelism

a. Know the Parallel Postulate and its implications, and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees)
b. Know that variants of the Parallel Postulate produce non-Euclidean geometries (e.g., spherical, hyperbolic)

(Mathematics Content Standards for California Public Schools, Algebra I: 8.0, 24.0; Geometry: 1.0-3.0, 7.0, 13.0)

2.2 Plane Euclidean Geometry

a. Prove theorems and solve problems involving similarity and congruence
b. Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, Triangle Inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse)
c. Understand, apply, and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle)
d. Justify and perform the classical constructions (e.g., angle bisector, perpendicular bisector, replicating shapes, regular n-gons for n equal to 3, 4, 5, 6, and 8)
e. Use techniques in coordinate geometry to prove geometric theorems

(Mathematics Content Standards for California Public Schools, Grade 6, Algebra and Functions: 2.0, 3.0; Measurement and Geometry: 2.0; Grade 7, Measurement and Geometry: 1.0-3.0; Algebra I: 8.0, 24.0; Geometry: 1.0-6.0, 8.0-16.0, 18.0-21.0; Algebra II: 16.0, 17.0; Trigonometry: 12.0-14.0, 18.0, 19.0; Mathematical Analysis: 5.0)

2.3 Three-Dimensional Geometry

a. Demonstrate an understanding of parallelism and perpendicularity of lines and planes in three dimensions
b. Understand, apply, and justify properties of three-dimensional objects from an advanced standpoint (e.g., derive the volume and surface area formulas for prisms, pyramids, cones, cylinders, and spheres)

(Mathematics Content Standards for California Public Schools, Grade 6, Measurement and Geometry: 1.0; Grade 7, Measurement and Geometry: 2.0; Algebra I: 24.0; Geometry: 2.0, 3.0, 12.0, 17.0; Mathematical Analysis: 5.0)

2.4 Transformational Geometry

a. Demonstrate an understanding of the basic properties of isometries in two- and three-dimensional space (e.g., rotation, translation, reflection)
b. Understand and prove the basic properties of dilations (e.g., similarity transformations or change of scale)

(Mathematics Content Standards for California Public Schools, Geometry: 11.0, 22.0)

Domain 3. Number Theory

Candidates demonstrate an understanding of the number theory and a command of the number sense contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of number theory and its underlying structures, candidates have a deep conceptual knowledge. They prove and use properties of natural numbers. They formulate conjectures about the natural numbers using inductive reasoning, and verify conjectures with proofs.

3.1 Natural Numbers

a. Prove and use basic properties of natural numbers (e.g., properties of divisibility)
b. Use the Principle of Mathematical Induction to prove results in number theory
c. Know and apply the Euclidean Algorithm
d. Apply the Fundamental Theorem of Arithmetic (e.g., find the greatest common factor and the least common multiple, show that every fraction is equivalent to a unique fraction where the numerator and denominator are relatively prime, prove that the square root of any number, not a perfect square number, is irrational)

(Mathematics Content Standards for California Public Schools, Grade 6, Number Sense: 2.0; Grade 7, Number Sense: 1.0; Algebra I: 1.0, 2.0, 12.0, 24.0, 25.0; Geometry: 1.0; Algebra II: 21.0, 23.0, 25.0; Mathematical Analysis: 3.0)

Domain 4. Probability and Statistics

Candidates demonstrate an understanding of the statistics and probability distributions for advanced placement statistics contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of probability and statistics and their underlying structures, candidates have a deep conceptual knowledge. They solve problems and make inferences using statistics and probability distributions.

4.1 Probability

a. Prove and apply basic principles of permutations and combinations
b. Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles)
c. Use and explain the concept of conditional probability
d. Interpret the probability of an outcome
e. Use normal, binomial, and exponential distributions to solve and interpret probability problems

(Mathematics Content Standards for California Public Schools, Grade 6, Statistics, Data Analysis, and Probability: 3.0; Algebra II: 18.0-20.0; Probability and Statistics: 1.0-4.0; Advanced Probability and Statistics: 1.0-4.0, 7.0, 9.0, 17.0, 18.0)

4.2 Statistics

a. Compute and interpret the mean, median, and mode of both discrete and continuous distributions
b. Compute and interpret quartiles, range, variance, and standard deviation of both discrete and continuous distributions
c. Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results
d. Know the method of least squares and apply it to linear regression and correlation
e. Know and apply the chi-square test

(Mathematics Content Standards for California Public Schools, Grade 6, Statistics, Data Analysis, and Probability: 1.0, 2.0; Grade 7, Statistics, Data Analysis, and Probability: 1.0; Probability and Statistics: 5.0-7.0; Advanced Probability and Statistics: 4.0-6.0, 8.0, 10.0-13.0, 15.0-17.0, 19.0)

Domain 5. Calculus[1]

Candidates demonstrate an understanding of the trigonometry and calculus contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of trigonometry and calculus and their underlying structures, candidates have a deep conceptual knowledge. They apply the concepts of trigonometry and calculus to solving problems in real-world situations.

5.1 Trigonometry

a. Prove that the Pythagorean Theorem is equivalent to the trigonometric identity $\sin^2 x + \cos^2 x = 1$ and that this identity leads to $1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \csc^2 x$
b. Prove the sine, cosine, and tangent sum formulas for all real values, and derive special applications of the sum formulas (e.g., double angle, half angle)
c. Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems)
d. Know and apply the definitions and properties of inverse trigonometric functions (i.e., arcsin, arccos, and arctan)
e. Understand and apply polar representations of complex numbers (e.g., DeMoivre's Theorem)

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0; Geometry: 3.0, 14.0, 18.0, 19.0; Algebra II: 24.0, 25.0; Trigonometry: 1.0-6.0, 8.0-11.0, 19.0; Mathematical Analysis: 1.0, 2.0; Calculus: 18.0, 20.0)

5.2 Limits and Continuity

a. Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit
b. Show that a polynomial function is continuous at a point
c. Know and apply the Intermediate Value Theorem, using the geometric implications of continuity

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0; Geometry: 3.0; Algebra II: 1.0, 15.0; Mathematical Analysis: 8.0; Calculus: 1.0-4.0)

5.3 Derivatives and Applications

a. Derive the rules of differentiation for polynomial, trigonometric, and logarithmic functions using the formal definition of derivative

[1] *Domain 5, Calculus, does not apply to requirements for the Foundational-level Credential.*

b. Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton's method, and instantaneous rate of change)
c. Interpret both continuous and differentiable functions geometrically and analytically and apply Rolle's Theorem, the Mean Value Theorem, and L'Hopital's rule
d. Use the derivative to solve rectilinear motion, related rate, and optimization problems
e. Use the derivative to analyze functions and planar curves (e.g., maxima, minima, inflection points, concavity)
f. Solve separable first-order differential equations and apply them to growth and decay problems

(Mathematics Content Standards for California Public Schools, Algebra I: 5.0-8.0, 10.0, 11.0, 13.0, 21.0, 23.0; Geometry: 3.0; Algebra II: 1.0, 9.0, 10.0, 12.0, 15.0; Trigonometry: 7.0, 15.0-19.0; Mathematical Analysis: 5.0, 7.0; Calculus: 1.0, 4.0-12.0, 27.0)

5.4 Integrals and Applications

a. Derive definite integrals of standard algebraic functions using the formal definition of integral
b. Interpret the concept of a definite integral geometrically, numerically, and analytically (e.g., limit of Riemann sums)
c. Prove the Fundamental Theorem of Calculus, and use it to interpret definite integrals as antiderivatives
d. Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0; Geometry: 9.0; Calculus: 13.0-23.0)

5.5 Sequences and Series

a. Derive and apply the formulas for the sums of finite arithmetic series and finite and infinite geometric series (e.g., express repeating decimals as a rational number)
b. Determine convergence of a given sequence or series using standard techniques (e.g., Ratio, Comparison, Integral Tests)
c. Calculate Taylor series and Taylor polynomials of basic functions

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0, 25.0; Algebra II: 21.0-23.0; Mathematical Analysis: 8.0; Calculus: 23.0-26.0)

Domain 6. History of Mathematics[2]

Candidates understand the chronological and topical development of mathematics and the contributions of historical figures of various times and cultures. Candidates know important mathematical discoveries and their impact on human society and thought. These discoveries

[2] *Domain 6, History of Mathematics, does not apply to requirements for the Foundational-level Credential.

form a historical context for the content contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999; e.g., numeration systems, algebra, geometry, calculus).

6.1 Chronological and Topical Development of Mathematics

a. Demonstrate understanding of the development of mathematics, its cultural connections, and its contributions to society
b. Demonstrate understanding of the historical development of mathematics, including the contributions of diverse populations as determined by race, ethnicity, culture, geography, and gender

Part II: Subject Matter Skills and Abilities Applicable to the Content Domains in Mathematics

(All elements of Part II apply to both the Single Subject Credential in Mathematics and the Single Subject Credential in Foundational Mathematics.)

Candidates for Single Subject Teaching Credentials in mathematics use inductive and deductive reasoning to develop, analyze, draw conclusions, and validate conjectures and arguments. As they reason, they use counterexamples, construct proofs using contradictions, and create multiple representations of the same concept. They know the interconnections among mathematical ideas, and use techniques and concepts from different domains and sub-domains to model the same problem. They explain mathematical interconnections with other disciplines. They are able to communicate their mathematical thinking clearly and coherently to others, orally, graphically, and in writing, through the use of precise language and symbols.

Candidates solve routine and complex problems by drawing from a variety of strategies while demonstrating an attitude of persistence and reflection in their approaches. They analyze problems through pattern recognition and the use of analogies. They formulate and prove conjectures, and test conclusions for reasonableness and accuracy. They use counterexamples to disprove conjectures.

Candidates select and use different representational systems (e.g., coordinates, graphs). They understand the usefulness of transformations and symmetry to help analyze and simplify problems. They make mathematical models to analyze mathematical structures in real contexts. They use spatial reasoning to model and solve problems that cross disciplines.

(Mathematics Content Standards for California Public Schools, Grade 6, Mathematical Reasoning: 1.0-3.0; Grade 7, Mathematical Reasoning: 1.0-3.0)

Appendix B

Mathematics Courses Offered in California Public Schools

Once students leave the self-contained or core classrooms of the earlier grades, they typically follow the sequence of mathematics courses shown below, with students planning to attend college or major in the sciences or mathematics completing more of the sequence.

Algebra I → Geometry → Algebra II/Trigonometry
→ Mathematical Analysis → Linear Algebra → Calculus

An alternative approach to the mathematics course sequence that is in place in some districts and is supported by the Common Core State Standards is presented here:

Mathematics I→ Mathematics II→ Mathematics III
→ Mathematical Analysis → Linear Algebra → Calculus

Probability and Statistics and Advanced Placement Statistics courses are also taught in some high schools. Traditionally, students enroll in Algebra I in 8th or 9th grade, although advanced 7th grade students may be recommended to take Algebra I. Because the California High School Exit Examination (CAHSEE) includes a small amount of Algebra material, a variety of classes has been developed across the state to provide students an opportunity to learn all of the material tested by the CAHSEE.

Staff retrieved data from the California Department of Education's DataQuest, http://dq.cde.ca.gov/dataquest/ to create Table 1, which provides several kinds of information. First, it identifies the complete universe of mathematics classes taught in California's public schools during 2010-11. The table also identifies the number of students who enrolled in each class and the number of schools that provided those classes. Overall, there were 3,035,757 students enrolled in 121,503 mathematics classes in 2010-11.

Rows that are shaded in light blue identify classes that can be taught by an individual with either a FLM or mathematics credential. Those classes range from Math 7 to Probability and Statistics and include general mathematics, algebra, geometry, probability and statistics, and consumer mathematics. The total number of classes is 103,570.

A literal interpretation of the FLM authorization permits individuals with FLM credential holders to also teach Intermediate Algebra (Algebra II), and Integrated Mathematics II. The total number of 2010-11 classes that could be taught by teachers with the FLM credential was 106,271 out of 121,503 mathematics classes.

The rows that are in blue shading with italics identify the classes about which there are serious concerns that the individual with a FLM may not have the content knowledge based on the current SMRs. It is unclear if the SMRs include the topics that are taught in these classes and, if so, are the topics addressed at the appropriate level. The authorization does not prohibit the FLM

teacher from teaching these courses. The classes that are shaded in peach are clearly beyond the FLM credential's authorization.

Table 1. Number of Students Enrolled in Mathematics Classes, the Number of Classes Offered, and the Number of Schools that Offer those Classes, 2010-11

Mathematics Classes at the Secondary Level	Number of Students Enrolled	Number of Classes	Number of Schools
Math 7 (grade 7 standards)	150,957	5,697	777
Algebra readiness	60,081	2,471	662
Other mathematics course	125,000	6,569	1,407
Math A	5,653	342	116
Math B	2,272	115	54
General math/basic math/vocational math	112,865	6,089	1,281
Consumer math/senior math	29,760	1,917	432
Remedial math/proficiency development	148,700	8,071	1,647
Pre-algebra	296,157	12,146	1,798
Beginning algebra/algebra I (one year course)	759,909	28,399	3,371
Integrated mathematics I college prep	12,363	487	110
Beginning Algebra Part 1(1st yr of 2 yr course)	69,281	3,691	748
Beginning Algebra Part 2 (2nd yr of 2 year course)	44,677	2,381	481
Geometry	465,790	17,063	2,267
Integrated mathematics II college prep	*4,649*	*184*	*57*
Intermediate algebra/algebra II	*292,780*	*9,972*	*1,341*
Probability and Statistics	*20,486*	*677*	*295*
Intermediate algebra and trigonometry	48,913	1,588	319
Trigonometry	24,644	876	259
Integrated mathematics III college prep	1,451	57	17
Solid geometry/trigonometry	6,956	220	44
Analytic geometry/pre-calculus	80,462	2,745	696
Adv algebra/adv geometry/symbolic logic/theory	17,584	616	203
Math analysis	38,457	1,274	306
Integrated Mathematics IV (college preparatory)	718	36	11
Calculus	7,792	346	174
Modern abstract algebra	38	2	2
Linear Algebra	485	17	5
Mathematics Totals	**3,035,757**	**121,503**	**20,024**

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CAMTE
California Association of Mathematics Teacher Educators

Professionals providing preservice and inservice education for K-12 mathematics teachers

www.camte.org

California Commission on Teacher Credentialing
1900 Capitol Avenue
Sacramento, CA 95811-4213

Dear Commissioners:

The California Association of Mathematics Teacher Educators (CAMTE), a professional organization of those involved in the preparation and professional growth of K-12 teachers of mathematics, believe that the subject matter requirements for the Single Subject Credential in Foundational-Level Mathematics are inadequate considering the classes those teachers are authorized to teach (i.e., Algebra I, Geometry, and Algebra II). As stated by Joanne Rossi-Becker in CAMTE's April 2011 memo to the Commission , "there is overwhelming consensus [among the mathematics teacher preparation community] that the FLM is missing the target with regard to its intended goal…" which was to increase the number of credentialed teachers competent to teach the single subject mathematics classes that are most frequently taken by students.

Colleagues in single subject credential programs report that credential students who had established subject matter competence via the California Subject Exams for Teachers (CSET) often have inadequate content knowledge to be effective single subject mathematics teachers. Subsequent data collected by CAMTE members support these concerns. It appears that many of those passing the CSET, especially at the Foundational Level, have taken little mathematics beyond high school courses. A paper, "An Initial Investigation into the Mathematical Background of Those Who Pass the CSET for Mathematics", containing this data is under review and will soon be published.

With the implementation of the new California Common Core Content Standards for Mathematics (CaCCCS-M), our concerns take on an added urgency. It is clear to us that the content knowledge needed to successfully implement the CaCCCS-M is different, and richer, than that measured by the current CSET. Students of teachers with limited mathematical knowledge will have little chance of mastering the new standards.

For these reasons, CAMTE requests that the Commission reevaluate the Subject Matter Requirements (SMR's) upon which the CSET is based, the CSET itself, and the level of course that can be taught by a teacher holding a Foundational Credential. CAMTE is ready and willing to assist in this work.

Respectfully submitted,

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Presentation to the California Commission on Teacher Credentialing • Sacramento, CA • 14 April 2011

Statement prepared by the CAMTE Advocacy Committee and delivered by CAMTE President Dr. Joanne Rossi Becker

Good morning, I am Joanne Rossi Becker, professor of mathematics at SJSU and President of the CA Association of Mathematics Teacher Educators (CAMTE). CAMTE represents professionals who provide pre-service and in-service education for mathematics teachers at all levels, and we appreciate the opportunity to share our thoughts about this agenda item.

There is overwhelming consensus that the FLM is missing the target with regard to its intended goal--that is, credentialing more appropriately qualified math teachers. Currently a prospective teacher who earns an FLM through the exam route is required to pass CSET subtests I and II. There is general agreement among mathematicians and mathematics educators that the CSET subtests for math are inadequate in assessing a teacher's depth and breadth of mathematical experience and understanding. Early results of a study being conducted by Dr. Jorgen Berglund at Chico State University indicate that around 50% of those who earned an FLM credential on California State University campuses and met subject matter competence via CSET subtests had taken no more than one or two university-level mathematics courses, and all of those were lower division courses. It seems that there is extremely poor alignment between what is required to pass CSET subtests I and II and what the CTC appropriately requires of the subject matter preparation programs.

Given this situation, we do believe that the Commission's proposal to reduce the number of semester units required in a Subject Matter Preparation program for the Foundational-Level Mathematics credential is a step forward in order to encourage campuses to be able to develop programs that encourage course-taking rather than test-taking for the credential. We just want to make sure that these programs are mathematically robust . It would be challenging to create a program with only 20 units of mathematics that would contain all the mathematics that an FLM credential holder should be taking in order to teach the extent of mathematics that she or he would be credentialed to teach.

Thus the second--and related--major concern that mathematics teacher educators have with the FLM credential is the level of mathematics that an FLM teacher is allowed to teach. As the FLM currently exists, a credential holder is authorized to teach through Algebra II. This is the course that completes the state's A-G requirements in mathematics for high school students who plan to attend college. It is extremely troublesome to many mathematics teacher educators in the state that a teacher as under-prepared as some of those currently earning an FLM credential could be teaching such an important and advanced course. There would be far less resistance to the FLM credential by mathematics teacher educators if the instructional authorization were limited to

Algebra I or Geometry. This more limited scope would be consistent with the authorization provided by the Subject Matter Authorization in Introductory Mathematics, an authorization that requires 32 semester units of mathematics coursework, compared to the FLM's minimum of 20 semester units of mathematics. It seems a bit incongruous for an authorization with fewer required university mathematics courses to permit the credential holder to teach more advanced mathematics courses.

None of these issues is new to the Commission. Within the last two years, these issues have been discussed at Commission meetings, at panel meetings, and in conversations with CTC staff. At more than one Commission meeting during that same time period, the commissioners expressed concern about the confusing nature of the myriad math credentials and authorizations in the state, and noted that a panel should be created to try to make the entire authorization structure more coherent. CAMTE supports the creation of such a panel and the organization's members would be eager to participate.

We know that the Commission shares our desire to credential the most qualified teachers of mathematics possible. We are pleased that we in CAMTE have begun a dialogue with the Commission and CCTC staff and we look forward to strengthening these lines of communication. We are confident that together we will find ways to improve the subject matter competence of those earning Single Subject Teaching Credentials in Mathematics.